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FINAL REPORT

Title of Project: Statistical Inference in Stochastic Processes

Period of Time: September 15, 1962 - June 30, 1967.

AROD Project Number: 3574-M

Grant Number: DA-ARO(D)-31-124-G383

Author of Report: Howard G. Tucker, Principal Investigator

Total personnel employed on this grant were the following graduate students:

Daniel G. Martinez

Howard H. Stratton, Jr.

Marllyn T. Boswell

Barthel Wayne Huff

Jesse David Mason

Joseph J. Sroka

Advanced degrees received:

(i) Daniel G. Martinez received his M.A. in Mathematics in 1963.

(ii) Howard H. Stratton, Jr. received his Ph.D. in Mathematics in June, 1965.

Title of dissertation: <u>On Dimension of Support for Stochastic Processes with Independent Increments</u>.

(iii) Marllyn T. Boswell received his Ph.D. in Mathematics In February, 1965. Title of dissertation: <u>Estimating and Testing Trend in a Stochastic Process of Poisson Type</u>.

5

(iv) Barthel Wayne Huff has enough results at this date for a doctoral dissertation entitled: <u>On Subordination of Stochastic Processes</u>.

(v) Jesse David Mason received his M.A. in Mathematics in January, 1966. He is in the middle of his doctoral dissertation on refinements of the Glivenko-Cantelli theorem connected with problems involving mixtures.

(vi) Joseph J. Sroka received his M.A. in Mathematics on December, 1966. He has not yet begun his doctoral dissertation.

PUBLICATIONS:

The following two papers which acknowledge support from this grant have already appeared in print:

[1] Howard E. Stratton, Jr. and Howard G. Tucker, "Limit distributions of a branching stochastic process," Annals of Mathematical Statistics, 35(1964), pp. 557-565.

In this paper the limiting distribution was obtained for a branching process of rather general form in which the initial size of the population was made to increase to infinity and other parameters were made to vary as a function of the initial size of the population. The limiting distribution is that of an integer-valued stochastic process with stationary independent increments. (A more complete resume of this was included in progress reports numbers 2 and 3.)

[2] B. W. Huff and W. A. Kirk, "A note on equilateral metric triples," American Mathematical Monthly, 74(1967), pp. 288-290.

This paper improves results, appearing in Distance Geometry by Leonard M. Blumenthal, from sufficient to best possible. These show that equilateral metric triples must exist whenever certain distance relations hold between a metric segment and a point not on the segment. Examples show that the constants appearing in the expressions are best possible.

The following paper, acknowledging suppo.t of this grant, has been submitted for publication to Transactions of the American Mathematical Society, but no referee's report has been received yet:

[3] Howard H. Stratton, Jr., "On dimension of support for stochastic processes with independent increments."

In this paper a characterization is obtained of the dimension of support of the continuous singular component of the distribution functions of a stochastic process with independent increments. By definition, if G is a bounded, continuous singular, nondecreasing function over $(-\infty,+\infty)$, then by the dimension of support of G one means

$$\dim \operatorname{supp} G = \{\dim C \mid \int_C dG = G(+\infty) - G(-\infty), \text{ all Borel sets } C\},$$

where dim C means the Hausdorff dimension of C. A special easy-to-state case of the result follows: if

$X(t)$ is a stochastic process with stationary independent increments, if $G_t(x)$ is the continuous singular component of $P[X(t) \le x]$, and if $\varphi(t) = \dim \operatorname{supp} G_t$, then (i) $0 \le \varphi(t) \le 1$, and (ii) φ is lower semi-continuous except at a countable set of points. Conversely, if I is any interval whose left endpoint is 0, and if φ is any function over I staisfying (i) and (ii), then there is a stochastic process with stationary independent increments, $X(t)$, such that $F_t(x) = P[X(t) \le x]$ is absolutely continuous for $t \in (0,\infty)\backslash I$ and such that the dimension of support of the continuous singular component of F_t for all $t \in I$ is $\varphi(t)$.

OTHER ACTIVITIES

The following topics are the principal ones investigated by the graduate students employed on this grant:

(1) Hypothesis testing for Poisson-like processes.

(2) Hilbert space methods in time series analysis.

(3) Limit distributions of branching processes.

(4) Hausdorff dimension in stochastic processes.

(5) Sufficient statistics for stochastic processes.

(6) Absolute continuity and orthogonality of stochastic processes.

(7) Subordination of stochastic processes.

(8) Slowly varying functions in probability.

(9) Mixture problems and the Glivenko-Cantelli Theorem.

(10) Random power series.

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Tucker, Howard G.

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13. ABSTRACT

The following topics are the principal ones investigated under this grant: (1) Hypothesis testing for Poisson-like processes. (2) Hilbert space methods in time series analysis. (3) Limit distributions of branching processes. (4) Hausdorff dimension in stochastic processes. (5) Sufficient statistics for stochastic processes. (6) Absolute continuity and orthogonality of stochastic processes. (7) Subordination of stochastic processes. (8) Slowly varying functions in probability. (9) Mixture problems and the Glivenko-Cantelli Theorem. (10) Random power series.

14. KEY WORDS

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